HIGHER MATHEMATICS
MATHEMATICS

Paper 7 Probability \& Statistics 2 (S2)
May/June 2005
1 hour 15 minutes

## Additional materials: Answer Booklet/Paper

Graph paper
List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

1 Exam marks, $X$, have mean 70 and standard deviation 8.7. The marks need to be scaled using the formula $Y=a X+b$ so that the scaled marks, $Y$, have mean 55 and standard deviation 6.96. Find the values of $a$ and $b$.

2 Jenny has to do a statistics project at school on how much pocket money, in dollars, is received by students in her year group. She plans to take a sample of 7 students from her year group, which contains 122 students.
(i) Give a suitable method of taking this sample.

Her sample gives the following results.

$$
\begin{array}{lllllll}
13.40 & 10.60 & 26.50 & 20.00 & 14.50 & 15.00 & 16.50
\end{array}
$$

(ii) Find unbiased estimates of the population mean and variance.
(iii) Is the estimated population variance more than, less than or the same as the sample variance?
(iv) Describe what you understand by 'population' in this question.

3 A survey of a random sample of $n$ people found that 61 of them read The Reporter newspaper. A symmetric confidence interval for the true population proportion, $p$, who read The Reporter is $0.1993<p<0.2887$.
(i) Find the mid-point of this confidence interval and use this to find the value of $n$.
(ii) Find the confidence level of this confidence interval.

4 A study of a large sample of books by a particular author shows that the number of words per sentence can be modelled by a normal distribution with mean 21.2 and standard deviation 7.3. A researcher claims to have discovered a previously unknown book by this author. The mean length of 90 sentences chosen at random in this book is found to be 19.4 words.
(i) Assuming the population standard deviation of sentence lengths in this book is also 7.3, test at the $5 \%$ level of significance whether the mean sentence length is the same as the author's. State your null and alternative hypotheses.
(ii) State in words relating to the context of the test what is meant by a Type I error and state the probability of a Type I error in the test in part (i).

5 A clock contains 4 new batteries each of which gives a voltage which is normally distributed with mean 1.54 volts and standard deviation 0.05 volts. The voltages of the batteries are independent. The clock will only work if the total voltage is greater than 5.95 volts.
(i) Find the probability that the clock will work.
(ii) Find the probability that the average total voltage of the batteries of 20 clocks chosen at random exceeds 6.2 volts.

6 At a petrol station cars arrive independently and at random times at constant average rates of 8 cars per hour travelling east and 5 cars per hour travelling west.
(i) Find the probability that, in a quarter-hour period,
(a) one or more cars travelling east and one or more cars travelling west will arrive,
(b) a total of 2 or more cars will arrive.
(ii) Find the approximate probability that, in a 12 -hour period, a total of more than 175 cars will arrive.

7 The random variable $X$ denotes the number of hours of cloud cover per day at a weather forecasting centre. The probability density function of $X$ is given by

$$
\mathrm{f}(x)= \begin{cases}\frac{(x-18)^{2}}{k} & 0 \leqslant x \leqslant 24 \\ 0 & \text { otherwise }\end{cases}
$$

where $k$ is a constant.
(i) Show that $k=2016$.
(ii) On how many days in a year of 365 days can the centre expect to have less than 2 hours of cloud cover?
(iii) Find the mean number of hours of cloud cover per day.

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